**Mathematics 2 Exam: Calculus**

Differentiation and Integration

**Total Marks: 100**

**Instructions:**

* Part A covers Differentiation and is worth 50 marks.
* Part B covers Integration and is worth 50 marks.
* Show all your work for each problem.
* Calculators may be used unless otherwise specified.
* Answer each question in the space provided.
* The use of trigonometric functions in differentiation and integration is not required.

**Part A: Differentiation (50 Marks)**

**Question 1 [10 Marks]**

Find the first and second derivatives of the following functions:

( 2.5 Marks )

( 2.5 Marks )

( 2.5 Marks )

( 2.5 Marks )

**Question 2 [10 Marks]**

Differentiate the following functions using the implicit differentiation method:

( 2.5 Marks )

( 2.5 Marks )

( 2.5 Marks )

( 2.5 Marks )

**Question 3 [10 Marks]**

* Given the function , find the value of where the tangent is horizontal. (5 Marks )
* Given the function , find all the values of x where the tangent is horizontal. ( 5 Marks )

**Question 4 [10 Marks]**

* A rectangular box without a lid is to be made from a 12 cm by 20 cm piece of cardboard by cutting equal squares from each corner and folding up the flaps. Let the side length of each square cut be *x*. Write a function that expresses the volume of the box in terms of *x*. Then, find the value of *x* that maximizes the volume. ( 5 Marks )
* A piece of square metal sheet measuring 30 cm on each side is used to make an open box by cutting squares of side length from each corner and folding up the sides. Express the volume of the box as a function of , and find the value of that maximizes the volume. ( 5 Marks )

**Question 5 [10 Marks]**

Find the derivative of the functions. Also, determine the intervals where the function is increasing or decreasing.

( 5 Marks )

( 5 Marks )

**Part B: Integration (50 Marks)**

**Question 1 [10 Marks]**

Evaluate the following integrals:

( 2.5 Marks )

***Solution:***

The antiderivative of is .

This function will have a constant of integration, which we usually denote by C.

So the integral is =

( 2.5 Marks )

***Solution:***

The antiderivative of is .

Multiplying this by 3 gives

( 2.5 Marks )

***Solution:***

The antiderivative of is

So the integral is

( 2.5 Marks )

***Solution:***

The antiderivative of is

Multiplying this by 2 gives

**Question 2 [10 Marks]**

**Find the area under the curve of the function , between and . ( 5 Marks )**

***Solution:***

To find the area under the curve of the function between and , we need to compute the definite integral of from to .

The antiderivative of is . Evaluating this from 1 to 3 gives:

**Find the area under the curve of the function , between and . ( 5 Marks )**

***Solution:***

To find the area under the curve of the function between and , we need to compute the definite integral of from to .

The antiderivative of is . Evaluating this from to gives:

**Question 3 [10 Marks]**

**Compute the following integrals:**

**( 2.5 Marks )**

***Solution:***

The antiderivative of is .

This function will have a constant of integration, which we usually denote by C. So the integral is

= .

**( 2.5 Marks )**

***Solution:***

The antiderivative of is

This function will also have a constant of integration, C.

So, the integral is

**Question 4 [10 Marks]**

* A company's total cost function is , where x is the number of units produced. Find the fixed and the marginal cost when . ( 5 Marks )
* A company’s revenue function is , where is the number of units sold. Find the profit function , where represents the total cost function from the previous question. Calculate the profit when . ( 5 Marks )

***Solution:***

The total cost function is given by

The antiderivative of is

This function will have a constant of integration, which represents the fixed costs (since they don’t depend on x). We will call this constant C. So, the total cost function is

The marginal cost is the derivative of the total cost function, which is

Evaluating this at gives

The revenue function is given by

The antiderivative of is

This function will also have a constant of integration, but it doesn’t affect the profit function since it will be subtracted out. So the revenue function is .

The profit function is

Evaluating this at gives

**Question 5 [10 Marks]**

Evaluate the following definite integrals:

( 2.5 Marks )

***Solution:***

( 2.5 Marks )

***Solution:***

( 2.5 Marks )

***Solution:***

( 2.5 Marks )

***Solution:***

**Answer Scheme:**

**Part A: Differentiation**

**Question 1**

**Question 2**

**Question 3**

* Find and solve for .
* Find for a given function . Solve for t to find the points where the tangent to the curve is horizontal.

**Question 4**

* Volume function: . Find and solve for .
* Cut equal squares from each corner, side length of each square cut is . Express the volume of the box as a function . Find the value of that maximizes the volume.

**Question 5**

Determine intervals where and

**Part B: Integration**

**Question 1**

**Question 2**

Area under curve:

Area under the curve:

**Question 3**

**Question 4**

Fixed cost:

Marginal cost at

**Question 5**